

Another example: *How many two-person chess teams can a coach make if there are four chess players in the class?*

This counting problem can be solved either by (a) enumerating (listing) all the possibilities, or (b) computing by formula the number of *combinations* of four things taken two at a time.

Some counting problems are solved quickly and easily using such concepts as combinations and permutations often taught with probability. But, unless the numbers are too large for practical enumeration, listing all the possibilities also works.

Here are more examples of counting problems:

Example: *There are three chairs around a table. How many ways can two people be seated?*

This problem is easy enough to be solved by enumerating all the possibilities, but it is also solvable by computing the number of *permutations* of three things taken two at a time. Permutations are discussed in Chapter 3.

Example: *Six pigeons are to be put into four cages. If there must be at least one pigeon in each cage, how many ways can this be done?*

Example: *Anne has four skirts, three sweaters, and two pairs of shoes. How many skirt-sweater-shoe outfits can she put together?*

Example: *Using quarters, dimes, and pennies, how many ways can the sum of 48 cents be produced?*

Counting problems and their solution processes are discussed in detail in Chapter 3.

1.4 'Word' Problems

Problems conventionally called 'word' problems are by far the most common type found in algebra and science textbooks. Strictly speaking, a vast majority of all math problems are presented verbally, and hence are word problems. For example, the counting problems discussed in the preceding section are usually presented in words. However, when we refer in this book to 'word' problems ('word' in quotes), we exclude counting as well as pattern recognition, recreation, and proof problems. We mean the type that are commonly *called* word problems; that is, the ones that students (and others) dread and struggle with.

'Word' problems are *deductive* in the sense that the solutions are derived from the given information,

which is assumed to be true and accurate. We don't usually use the term 'deduction' in connection with these problems, however, because that term is so widely used in formal logic.

'Word' problems come in a wide variety of subject matter contexts, including at least the following:

- coins and money
- numbers
- age
- average
- volume, area, and perimeter
- interest
- mixtures
- ratio and proportion
- time-rate-distance
- other 'per' problems (e.g., words per page)
- geometry and shapes
- trigonometry
- graphs and functions.

Here is a simple **example:** *Three times a number is fourteen less than five times the number. What is the number?*

Other Examples of 'Word' Problems

Example: *A rectangular field is four times longer than it is wide. Its perimeter is 1000 yards. What is the area of the field?*

Example: *A car travelling at an average speed of 40 mph leaves home at 8 am. A second car travelling at an average speed of 50 mph along the same route leaves at 10 am. At what time does the second car catch up with the first?*

Example: *An equilateral triangle is inscribed inside a circle of radius 2 inches. The vertices of the triangle intersect the circle. What is the perimeter of the triangle in centimeters.*

Example: *A collection of fifteen coins — quarters, dimes, and pennies — is worth \$1.23. There are twice as many pennies as dimes. How many of each type coin are there?*

'Word' problems are solved by a four-phase process which, for lack of a better word, we call *analysis*. Literally, analysis means to 'take apart' and/or to 'study the parts' as a way of gaining understanding. In our analysis process, the 'taking apart' is done mostly in the second step where we translate the given verbal information into a symbolic representation.