Thanks for looking at our Optimization problem examples

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Table of the types of optimization problems solved in Chapter 5.

Types of Organization Problems Solved in Chapter 5

Optimization With One Variable

Example 5.1 - Play Yard
Example 5.2 - Maximize a^2b^2
Example 5.3 - Maximize Profits
Example 5.4 - Dog Kennels
Example 5.5 - Cylindrical Tank
Example 5.6 - Right Triangle
Example 5.7 - Rectangle
Example 5.8 - Shortest Time

Optimization With Two Variables

Example 5.9 - Two Symbolic Variables
Example 5.10 - Rectangular Tank (Cost)

Linear Programming

Example 5.10 - Solution at the Corners
Example 5.11 - Rectangular Tank

Below

Three sample problems are presented.
They have three levels of difficulty
• Easier
• Moderate
•Harder
Scroll or page down for the sample.s

Easy Optimization Example

Problem: A child's rectangular play yard is to be built next to the house. To make the three sides of the play-pen, twenty-four feet of fencing are available. What should be the dimensions of the sides to make a maximum area?

Solution:

__Preparation:__ Know the problem thoroughly

- Read the problem statement again, carefully.

A child's rectangular play yard is to be built next to the house. To make the three sides of the playpen, twenty-four feet of fencing are available. What should be the dimensions of the sides to make a maximum area?

- Restate the given information clearly

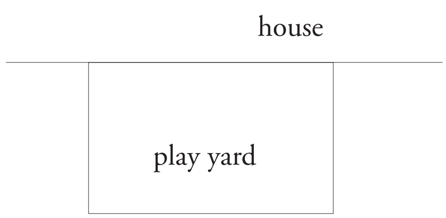
3 sided play yard - rectangular
24 feet of fencing available
Make area as large as possible.

- In words, write what is to be found

Dimensions of pen for maximum area

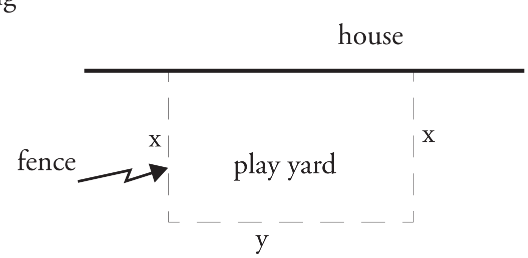
__Translation__ into an optimization formulation.

- __Drawing (As important as ever!)__



- __Define Symbols:__ (Also as important as ever!)

We can add some symbols (and units) to the drawing



- __Other Symbols__:

A = total area of yard = x y (sq. feet))
L = total length of fence (feet) = 24

- **Now the optimization formulation**:

(a) **Design variables:** x, y
(b) **Criterion Function:** Area = A = x y
(c) **Constraint:** 2 x + y = 24
or y = 24 - 2 x (1)

Note that the constraint (1), in effect, reduces the number of variables from two (x and y) to one: (x).

__**Application: Solution by Graphing**__

Construct a graph of the criterion function, A, and the (now) single design variable, x. A table of values is:

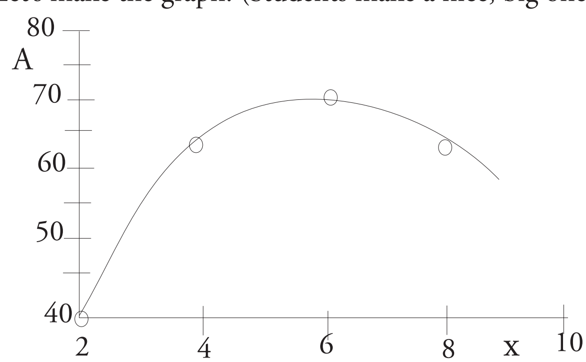
x (ft)	y = 24 -2x (ft)	ft^2
2	20	40
4	16	64
6	12	72
8	8	64

__Note:__ x and y are in feet. A is in ft^2.

We can see from just this much that the maximum is between x = 4 and x = 8, so we try some more values between 4 and 8:

x	y = 24 -2x	A = xy
4	16	64
5	14	70
6	12	72
7	10	70

Let's make the graph. (Students make a nice, big one.)



The symmetry around x = 6 leads us to suspect that that the optimum is at x = 6, where A = 72 ft^2 . We could verify that assumption with trials at x = 5.9 and 6.1. The maximum is at x = 6 and y = 12 ft.

Moderate Optimization Example

Problem: An open-top cylindrical tank with a volume of ten cubic feet is to be made from a sheet of steel. Find the dimensions of the tank that will require as little material used in the tank as possible.

Solution:

1 - Preparation Re-reading the problem statement and writing down what is given and what is to be found.

"An open-top cylindrical tank with a volume of ten cubic feet is to be made from a sheet of steel. Find the dimensions of the tank that will require as little material used in the tank as possible."

- Rewrite the given info

A cylindrical tank open at the top.

Made from as little sheet material as possible.

Volume is 10 ft^3 .

- To Find Dimension of the tank that minimizes the sheet material needed.

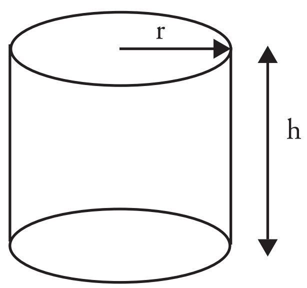
2 - Translation to an optimization problem:

"As little material as possible" translates to "as little outside surface area of the tank as possible." Or better: "minimum outside surface area."

- Drawings:

V = Volume = 10 ft^3

A = Area
$= \pi r^2 + 2 \pi r h$



- Symbols: Four definitions are in the drawing above.

Volume = V ft^3 . Surface Area = A ft^2

r = radius of tank (ft) h = height of tank (ft)

Formulation as an optimization problem

Design Variables: **r, radius (ft)**

h, height (ft)

The Criterion Function in this example is the sur- face area, A, since when it is minimum, the amoun of material used will be a minimum.

Criterion Function = A = $\pi r^2 + 2 \pi r h$

A Constraint in this case is the required volume.

Constraint: $V = \pi r^2 h = 10$ ft^3.

Or: $h = 10 / \pi r^2$

Now we have the optimization problem in mathematical form. In words, the math problem is the find the values of r and h that give a tank of minimum surface area with a volume of 10 ft^3.

3 - Application : Solution by Numerical Search.

We can almost always proceed by systematic numerical search.

In this example, there is really only a single unknown design variable because the constraint on the volume can be used. The table below shows a solution found by trying a value for r, computing h from the volume, and then computing the resulting surface area, A. This table was produced by using a spreadsheet program which is most convenient for such problems, but not really necessary in such a simple one-variable search.

One can start anywhere, but starting with r=1 seems a reasonable place. After the third trial, it is apparent that the minimum A is between r=1 and r=3.

Thus we plan to try r=1.5 and r=2.5. However, the result of r=1.5 reveals that the minimum A will be between r=1 and r=2. We continue with this process as shown.

Trial	r	$h = 10/\pi r^2$	$A = \pi r^2 + 20/r$
1	1	3.18	dec 23.14
2	2	0.796	22.56
3	3	0.35	inc 34.93
4	1.5	1.414	20.40
5	1.25	2.04	20.91
6	1.75	1.04	21.04
7	1.4	1.62	20.44
8	1.3	1.88	20.69
9	1.45	1.51	20.39
10	1.46	1.49	20.39
11	1.47	1.47	20.39
12	1.48	1.45	20.39

The optimum appears to be 20.39 ft^2, and it is found at a value of r approximately equal to 1.45 or 1.46 ft. To get more accuracy, we could refine the search.

Example 5.10 — Search for Two Variables

Problem:

Find the values of x and z (both > 0) that maximize

$$U = - x^2 + 10 x + xz - z^2 + 8z + 2$$

Solution:

Preparation:

- Re-read it and look at it closely:

Find the values of x and z (both > 0) that maximize

$$U = - x^2 + 10 x + xz - z^2 + 8z + 2$$

Note the two design variables x and z. Note, too, that there are no constraints that can be used to eliminate one of them.

- Rewrite the given info

$$U = - x^2 + 10 x + xz - z^2 + 8z + 2$$

- To Find: *Values of x and z that maximize U. (x and z are both >0.)*

Translation to an optimization problem:

> **Formulation as an optimization problem:**
>
> **Criterion function:**
> $U = - x^2 + 10 x + xz - z^2 + 8z + 2$
>
> **Design variables:** x, z
>
> **Constraints:** x, z positive

Application:

We will solve this using calculus.

(To solve the problem graphically requires three dimensions (x, z, U). There are methods for doing this but they are beyond our interest.)

$$\frac{\partial U}{\partial x} = (-2x + 10 + z) = 0$$

$$\frac{\partial U}{\partial z} = x - 2z + 8 = 0$$

Solving the above two equations for x and z gives:

x = 9.333 and z = 8.667

With these values, U = 83.33.

For More Information on

Solving Math Problems

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